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Noncircular Gears: Geometry and Visualization MODEL Development

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14. ABSTRACT
Circular gears are optimized to provide constant torque and speed ratio with low noise. In conventional automotive applications, a transmission provides the required power, torque, and speed settings using a finite arrangement of gear sets, which limits the range of transmission gear ratios, resulting in a loss of efficiency. However, other transmission configurations operate over a continuous range of gear ratios, providing a continuous range of output speeds while maintaining constant input shaft speed. Circular gears are not always practical for these configurations. One solution incorporates noncircular gears. The asymmetry of a noncircular gear pair results in instantaneously changing gear properties throughout gear rotation. This report presents a method for determining the required parameters to produce a noncircular gear pair of desired velocity and torque ratio distribution. Finally, to graphically depict the noncircular effects on gear motion, torque, and velocity distributions, a visualization model has been developed.

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Noncircular gear, gear geometry, gear analysis

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Introduction

Circular gears have been used in conventional power train applications for over a century. They are optimized to provide constant torque and speed ratio with low noise. In conventional automotive applications, the transmission provides the required power, torque, and speed settings using a finite arrangement of gear sets. This finite number limits the range of transmission gear ratios, resulting in a loss of efficiency. However, other configurations have been used for power transfer, such as the continuously variable transmission (CVT) or, more precisely, the infinitely variable transmission (IVT). These configurations operate over a continuous range of gear ratios, allowing for a multitude of output speeds while maintaining constant input shaft speed. Circular gears are not always practical for these types of applications, necessitating another method of power transfer. One solution is to incorporate noncircular gears.

Noncircular gears have proven effective for use in CVT/IVT applications since at least the 1970s, due to their effects on gear ratio and torque transfer (*1*). The asymmetry of the gear pair results in instantaneously changing gear properties throughout the rotation of the gears. This requires more in-depth analysis to determine the gear parameters required to achieve a certain profile. Indeed, the geometric considerations for ensuring proper tooth contact and alignment are well documented (*2*, *3*). Furthermore, the gear shapes, themselves, must be designed to achieve the desired velocity profile. This report presents a method for determining the required noncircular gear parameters to produce a desired velocity and torque ratio distribution. The steps of the methodology are introduced in detail. It is then demonstrated using an example from the literature. Finally, in order to better understand the noncircular effects on gear motion, torque, and velocity distributions, the results have been captured graphically through the development of a noncircular gear visualization model.

Methodology

In circular gear applications, the speed and torque ratios are constant throughout the gear rotation according to the well-established relationships in equation 1.

$$V_r = \frac{\omega_2}{\omega_1} = \frac{T_1}{T_2} = \frac{r_1}{r_2}, \tag{1}$$

where (V_r) is defined as the velocity profile between the driving gear and driven gear. The velocity profile is defined as the velocity or speed ratio of the gear pair over one revolution. It is also the inverse of the torque ratio.

For noncircular gears, the same relationships hold, but the ratios of equation 1 are dependent upon the angles of rotation of the gears, (θ_1) and (θ_2). Thus, equation 1 can be written as

$$V_r = \frac{\omega_2(\theta_2)}{\omega_1(\theta_1)} = \frac{T_1(\theta_1)}{T_2(\theta_2)} = \frac{r_1(\theta_1)}{r_2(\theta_2)}, \tag{2}$$

where the speed and torque ratios are functions of the angles of rotation. As will be developed, these relationships have a pronounced effect on the velocity profile.

In the case of circular gears, the angles of rotation are unimportant because they do not impact the ratios listed in equation 1. In noncircular gears, however, (θ_1) and (θ_2) drive the entire design of the gear geometry and must be calculated. Therefore, the desired velocity profile, (V_r), must be designed or known *a priori*. For purposes of this report, a velocity profile from the literature is assumed (*4*), and is depicted in figure 1. The angle of rotation on the *x*-axis belongs to the input gear (θ_1). It is the independent variable in the methodology and drives all aspects of gear pair motion.

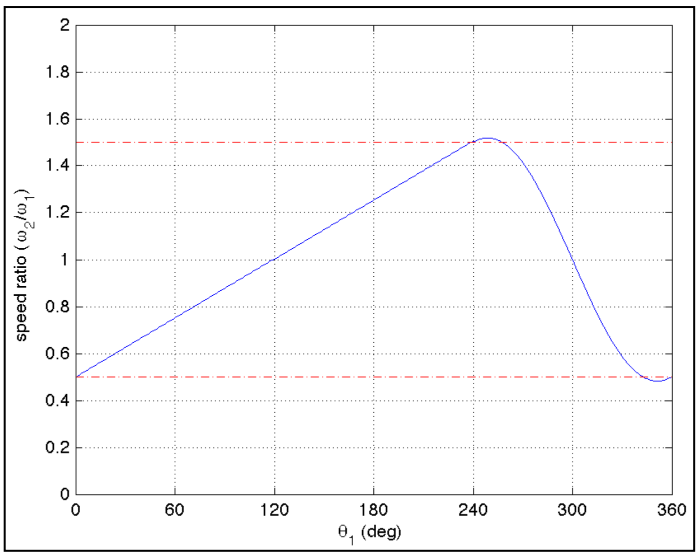

Figure 1. Velocity profile during one revolution.

The velocity profile has a desired minimum value of 0.5 and a desired maximum value of 1.5. These are indicated on figure 1 by the two dotted horizontal lines. The average velocity ratio, (V_{r_avg}), is thus 1.0. A value of 1.0 further indicates that both gears achieve a full rotation at the same time.

Figure 1 also shows two distinct areas of the profile: a linear portion and a nonlinear portion. The linear portion accounts for two-thirds of the rotation of Gear 1, as noted on the *x*-axis from 0–240°. From 240–360°, the ratio is nonlinear and sinusoidal. Note that the absolute maximum and minimum values of the velocity profile at the transition points between linear and nonlinear,

and vice versa, are slightly higher and lower, respectively, as a result of enforcing a constant slope of the velocity curve at the transition points.

The linear portion of the curve takes the familiar slope-intercept form of equation 3.

$$V(\boldsymbol{\theta}_1)_{r\_linear} = m\boldsymbol{\theta}_1 + V_{r\_\boldsymbol{\theta}_1=0} \tag{3}$$

The nonlinear relationship is slightly more complex. The desired function is a standard sinusoid according to the form of equation 4. The two transitions between linear and nonlinear portions must be smooth and continuous. This requires the linear and nonlinear functions to have the same speed ratios and the same slope at the points of transition.

$$V(\boldsymbol{\theta}_1)_{r\_nonlinear} = V_{r\_avg} + a\sin(\boldsymbol{\omega}\boldsymbol{\theta}_1 + \boldsymbol{\phi}) \tag{4}$$

Equation 4 contains three unknowns: the function amplitude (a), frequency (ω), and phase shift (ϕ). An iterative technique, enforcing the required boundary conditions, easily determines the values of these coefficients. Once the expressions of equations 3 and 4 have been determined, the velocity profile can be used to calculate the radii of the two gears and, then, their angles of rotation.

The radii of the two gears are continuously changing as the gears rotate. The radii can, therefore, be calculated as a function of the velocity profile. Assuming the length between the centers of the driving and driven gears as unity, $r_1+r_2=1$, and manipulating equations 1 and 2 yields equation 5:

$$r_1 = \frac{1}{\left(\frac{1}{V_r} + 1\right)} \tag{5}$$

Equation 5 is a nondimensional representation of radius, presented as a percent of the distance between gear centers. These results are presented in figure 2, where the y-axis below the curve represents the radius of the driving gear as a percentage of the center distance. The y-axis above the curve depicts the radius of the driven gear.

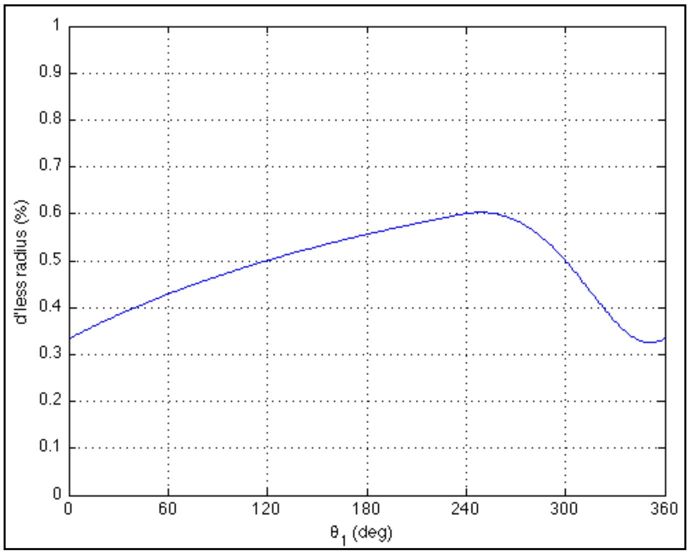

Figure 2. Dimensionless gear radius during one revolution.

The curve in figure 2 depicts the radial profile, which achieves the desired velocity profile of figure 1. There are two points where the non-dimensional radius equals 0.5, occurring at (θ_1) values of 120° and 300°. These points indicate that the radii of the driving and driven gears are both one-half the length between gear-shaft centers. The result is a speed ratio of one at these two points, as is confirmed in figure 1.

Returning to the discussion of the angles of rotation, (θ_1) and (θ_2), the relationship between these two is critical to developing the shape of the gear and for ensuring conjugacy of the gear teeth. For each value of (θ_1), there is a corresponding value of (θ_2), which will generate the profiles in figures 1 and 2. The values of (θ_2) as a function of (θ_1) can be determined by calculating the area under curve of the velocity profile in figure 1 (*4*), which is accomplished by integrating equations 3 and 4. The resulting expressions are given in equations 6 and 7. Using these expressions, the relationship between the rotations of the driving and driven gears can be determined for a full revolution.

$$\theta_2 = \int_0^{\theta_{1\_linear\_max}} \left(m\theta_1 + V_{r_{\theta_1}=0} \right) d\theta_1 \tag{6}$$

$$\theta_2 = \theta_{1\_linear\_max} + \int_{\theta_{1\_linear\_max}}^{360} a \sin\left(\omega\theta_1 + \phi \right) d\theta_1 \tag{7}$$

A convenient way to illustrate this relationship is to plot the values of (θ_1) and (θ_2) on the *x*-axis and *y*-axis, respectively. Plotting a reference line with a slope of unity provides a visual comparison of the angles of rotation. This is depicted in equation 3.

The correct interpretation of the curve in figure 3 is necessary to obtain any relevant information from it. Going back to figure 1, the velocity ratio is less than one between 0° and 120°. The driving gear is turning faster than the driven gear; thus, (θ_1) is growing "faster" than (θ_2). The

curve in figure 3 should correspondingly move south of the center-line. At 120°, the velocity ratio is equal to one, which results in the maximum deflection south of the center-line. Beyond 120°, the driven gear is rotating faster than the driving gear. This is indicated by the steepening slope of the curve and movement back toward the center-line. At 240°, the point of transition from linear to nonlinear velocity profile, the velocity ratio is still greater than one, such that (θ_2) is growing faster than (θ_1). Additionally, remembering that both gears reach a full rotation together, it holds true that both gears reach the transition point of 240° together, as well. This is indicated by the curve recovering the center-line at 240°. Soon after this point, the velocity ratio begins to decrease rapidly along the sinusoidal curve. However, the velocity ratio is still greater than one, causing the curve to continue moving north of the center-line. At 300°, the velocity ratio decreases to one, which again is the maximum deflection of the curve from the center-line. Beyond 300°, (θ_1) is once again growing faster than (θ_2). The curve moves southward until both angles meet at 360°, since both gears complete one revolution simultaneously.

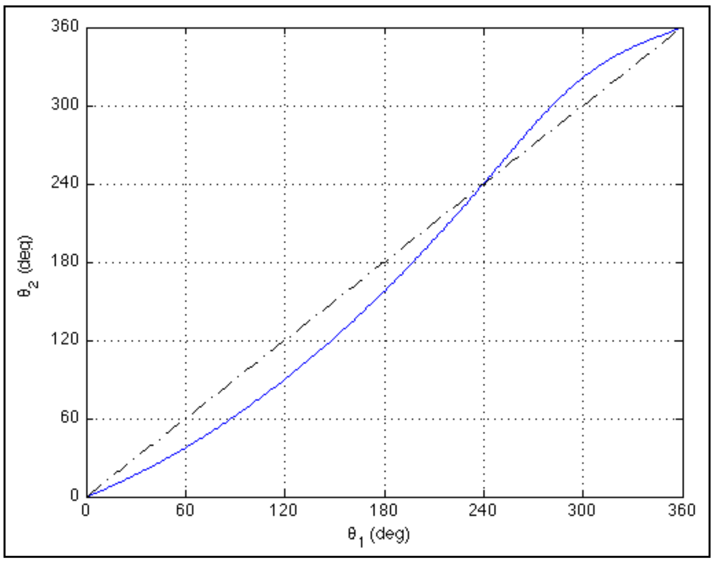

Figure 3. Gear rotation angle comparison.

The shapes of the two gears themselves can be determined using the relationships in equation 8.

$$
\begin{aligned}
x_{1i} &= Lr_{1i}\cos\left(\theta_{1i}\right)\\
y_{1i} &= Lr_{1i}\sin\left(\theta_{1i}\right)\\
x_{2i} &= Lr_{2i}\cos\left(180^\circ - \theta_{2i}\right)\\
y_{2i} &= Lr_{2i}\sin\left(180^\circ - \theta_{2i}\right)\\
i &= 1, 2, \ldots, n
\end{aligned}
\quad , \qquad (8)
$$

where (n) is the total number of theta segments per revolution. Plotting the results of equation 8 yields the geometry of the noncircular gear patterns, which are shown in figure 4.

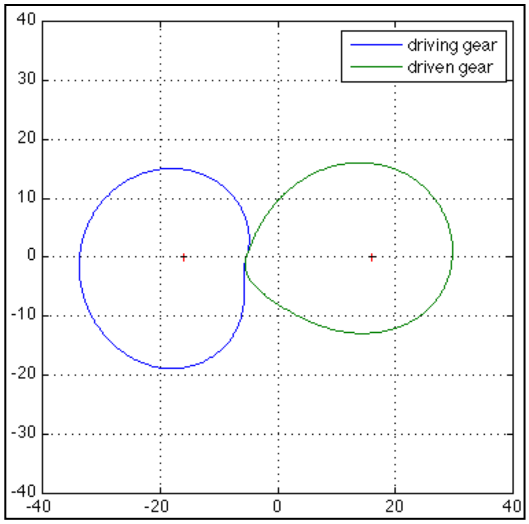

Figure 4. Noncircular gear dimensions.

From figure 4, the gear geometry calculated using equations 1–8 is clearly noncircular. The driving gear (left) is somewhat kidney-shaped. The driven gear (right) has an egg shape. As the gears rotate, they will always mesh at the pitch point, as conjugacy has been enforced throughout. However, unlike circular gears, the pitch point, itself, is not stationary. From figure 4, the pitch point slides along the line connecting the center-points of the two gear shafts.

The driven gear has one pointed "lobe," giving it the shape of an egg. This is a consequence of the average velocity ratio having a value of 1.0. The number of "lobes" of the driven gear is the inverse of its average velocity ratio. A velocity ratio of 0.5 will result in two lobes. Correspondingly, a value of 0.333 results in a driven gear with three lobes. These examples are easily calculated using this methodology and are presented in figures 5 and 6. Nevertheless, while it is theoretically possible to create a gear of several "lobes," the manufacturing and tooth meshing requirements constrain their feasibility in actual practice.

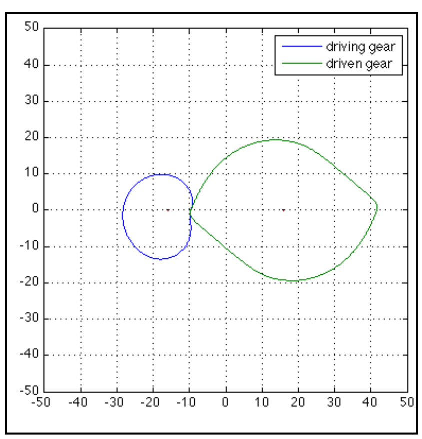

Figure 5. Two-lobe driven gear.

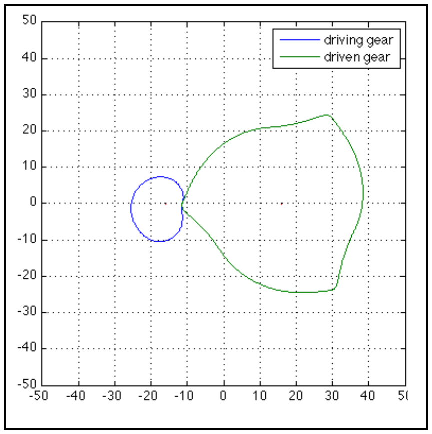

Figure 6. Three-lobe driven gear.

Visualization Model

One of the most difficult aspects of noncircular gears to grasp is how the asymmetry affects the gear properties of torque and speed through a complete revolution. A visualization tool has been developed to address this. The tool is a Matlab-based script capturing the rotational motion of the gears as well as speed, torque, and velocity ratio. The driving shaft speed input and the driven torque output are both held constant at unity. The driven shaft speed output and the driving torque input are calculated using equation 9.

$$
\begin{aligned}
T_1 &= T_2 \frac{1}{\eta} \frac{r_1}{r_2} \\
\omega_2 &= \omega_1 \frac{r_1}{r_2} \quad ,
\end{aligned}
\tag{9}
$$

where (η) represents the efficiency of the gear mesh. This can have a wide range of values depending upon the quality of the gear material, surface finish, manufacturing tolerances, etc. For highly precisioned gears, the value assumed is 99%.

Equation 8 yields the *x* and *y* coordinates of the stationary gear geometries. Equation 10 calculates the rotated *x* and *y* coordinates for the desired angles of rotation to create the rotational motion of the visualization model.

$$
\begin{aligned}
\begin{Bmatrix} x'_{1i} \\ y'_{1i} \end{Bmatrix} &= \begin{bmatrix} \cos\theta_{1i} & \sin\theta_{1i} \\ -\sin\theta_{1i} & \cos\theta_{1i} \end{bmatrix} \begin{Bmatrix} x_{1i} \\ y_{1i} \end{Bmatrix} \\
\begin{Bmatrix} x'_{2i} \\ y'_{2i} \end{Bmatrix} &= \begin{bmatrix} \cos(-\theta_{2i}) & \sin(-\theta_{2i}) \\ -\sin(-\theta_{2i}) & \cos(-\theta_{2i}) \end{bmatrix} \begin{Bmatrix} x_{1i} \\ y_{1i} \end{Bmatrix}
\end{aligned}
\tag{10}
$$

Figures 7–10 provide four sequential stages of gear rotation produced using the visualization model, along with the corresponding instantaneous values of torque, speed, and speed ratio.

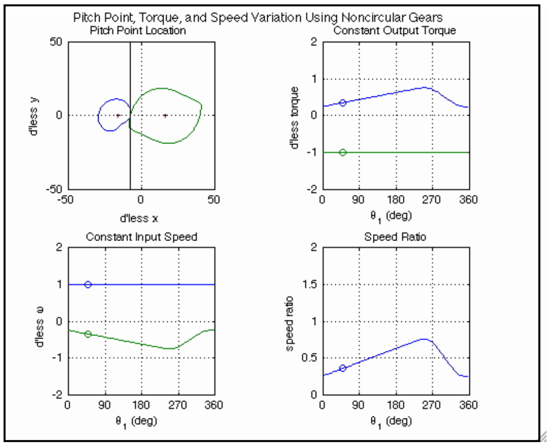

Figure 7. Visualization model capture 1.

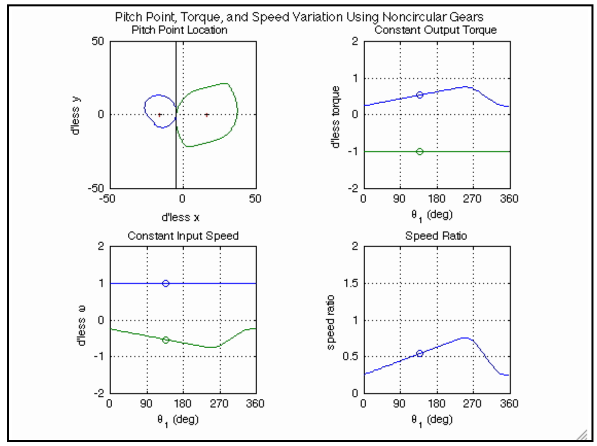

Figure 8. Visualization model capture 2.

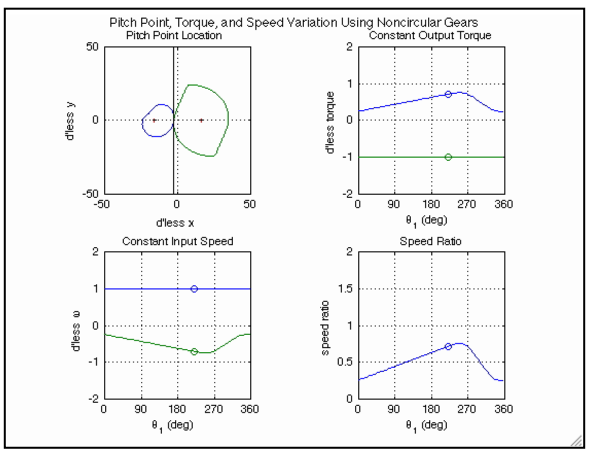

Figure 9. Visualization model capture 3.

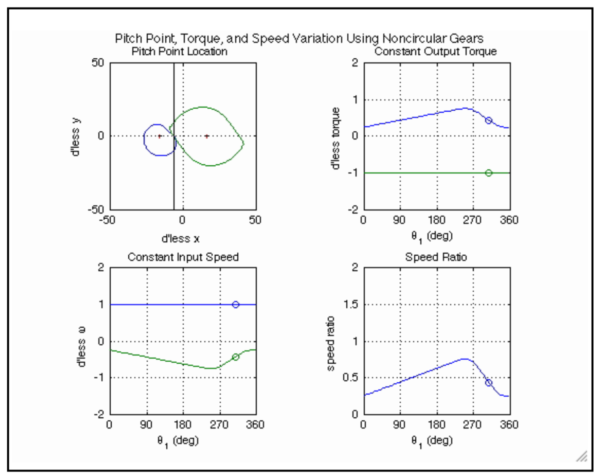

Figure 10. Visualization model capture 4.

Conclusion

This report has presented a methodology for determining the geometric and dynamic parameters of noncircular gears in rotation. The velocity profile was introduced as the parameter that ultimately drives the resulting design of the gear. The linear and nonlinear velocity profile functions were determined using an iterative solution method. The radii of the gears were determined. The relationship between the angles of the driving and driven gears was calculated. This resulted in the calculation of the gear geometries. Finally, the methodology was automated into a visualization model to graphically show how the gear parameters fluctuate using

noncircular gears. This type of analysis forms the basis for analyzing more complex driven systems using noncircular gears, such as the IVT.

References

1. Ferguson, R.; Daws, L.; Kerr, J. The Design of a Stepless Transmisssion Using Noncircular Gears. *Machine and Mechanism Theory* **1975**, *10* (6), 467–478.
2. Litvin, F.; Fuentes-Aznar, A. *Noncircular Gears: Design and Generation*; New York: Cambridge University Press; 2009.
3. Litvin, F.; Fuentes, A. *Gear Geometry and Applied Theory*; 2nd Ed. New York: Cambridge University Press; 2004.
4. Kerr, M. All Gear Infinitely Variable Transmission, U.S. Patent 6,849,023, 2005.

Nomenclature

m	slope of linear velocity profile
r_i	gear radius
T_i	gear torque
V_r	velocity ratio
θ_i	gear angle of rotation
ω_i	gear rotational speed

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